

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 1}{x^2 + 5} \xrightarrow{\frac{1}{x^2}} \frac{3}{1} \rightarrow 3$$

$$\lim_{x \rightarrow \infty} \frac{3 + x^{\frac{1}{2}}}{1 + \frac{5}{x^2}} = \frac{3 + 0}{1 + 0} = 3$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(x)} = \frac{\cos x}{1} = 1$$

$$\lim_{x \rightarrow \infty} \frac{5x + 2}{x^2 + 1} = \frac{\frac{5}{x} + \frac{2}{x^2}}{1 + \frac{1}{x^2}} = 0$$

$$\frac{5x + 2}{x^2 + 1} = \frac{\frac{d}{dx}(5x + 2)}{\frac{d}{dx}(x^2 + 1)} = \frac{5}{2x}$$

$$\lim_{x \rightarrow \infty} \left(\frac{5x + 2}{x^2 + 1} = 0 \right) \quad \frac{d}{dx}(x^2 + 1) = 2x$$

$$\lim_{x \rightarrow \infty} \frac{5x + 2}{x^2 + 1} = \frac{\frac{5}{2x} + \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}} = \frac{\frac{5}{x} + \frac{2}{x^2}}{1 + \frac{1}{x^2}}$$

$$\frac{5}{x} = 0, \quad \frac{2}{x^2} = 0, \quad \frac{1}{x^2} = 0 \Rightarrow \frac{0 + 0}{1 + 0} = 0$$

$$\text{Limit } x \rightarrow 3 \quad \frac{x^2 - 9}{x - 3} = \frac{(x - 3)(x + 3)}{(x - 3)}$$

$$x + 3 = \quad x \neq 3$$

$$\text{Limit } x = 3$$

$$x + 3 = 3 + 3 = \boxed{6}$$

$$\text{Limit } x \rightarrow \infty \left(\frac{1}{2} + \frac{6}{x} + \dots \right) = \boxed{\frac{1}{2}}$$

$$\frac{x - x - 1}{x^2}$$

$$\text{Limit } x \rightarrow \infty \quad \frac{4x^3 - x}{2x^3 + 5} = \frac{3x \left(4 - \frac{1}{x^2} \right)}{\left(2 + \frac{5}{x^3} \right)}$$

$$\frac{4 - \frac{1}{x^2}}{2 + \frac{5}{x^3}}$$

$$\frac{4 = 0}{2 + 0} = \frac{4}{2} = \boxed{2}$$

$$\text{Limit } x = 2 \quad \frac{x^2 + 4}{x - 2} = \frac{x^2}{x} = \boxed{2}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2} = \frac{d}{dx} \left(\frac{1 - \cos(x)}{\frac{d}{dx}(x^2)} \right)$$

$$= \frac{\sin(x)}{\frac{d}{dx}(2x)}$$

$$\frac{\sin(x)}{2x} = \frac{\cos(x)}{2} = \frac{\cos 0}{2} = \frac{1}{2}$$

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \frac{0}{0}$$

$$\frac{x^3 - 1}{x} = 1$$

$$\lim_{x \rightarrow \infty} \left(\frac{x^2 + 1}{x^2} \right) =$$

$$\frac{x^2 \left(1 + \frac{1}{x^2} \right)}{x^2} = 1 + \frac{1}{x}$$

$$1 + 0 = 1$$