$f(x) = x^2 + 4x + 2$

$a = 0$

$b = 4$

$$\int_0^4 (x^2 + 4x + 2)\, dx$$

$$\int_a^b f(x)\, dx = \lim_{n \to \infty} \sum_{k=1}^{n} f(a + k\Delta x)\Delta x \qquad \Delta x = \frac{b-a}{n}$$

Calculamos Δx

El ancho de cada subintervalo Δx se calcula con la formula

$$\Delta x = \frac{b-a}{n} = \frac{4-0}{n} = \frac{4}{n}$$

Calculamos $f(a + k\Delta x) =$

$f\left(0 + k\left(\frac{4}{n}\right)\right) = f\left(0 + \frac{4k}{n}\right)$

Evaluamos la función en

$f = \frac{4k}{n}$

Sustituimos la función

$f\left(\frac{4k}{n}\right) = \left(\frac{4k}{n}\right)^2 + 4\left(\frac{4k}{n}\right) + 2$

Calculamos cada parte

$\left(\frac{4k}{n}\right)^2 = \frac{16k^2}{n^2}$

$4 \cdot \frac{4k}{n} = \frac{16k}{n}$

Constante: 2

Entonces:

$f\left(\frac{4k}{n}\right) = \frac{16k^2}{n^2} + \frac{16k}{n} + 2$

Sustituimos en la suma de Riemann

$\sum_{k=1}^{n} f\left(\frac{4k}{n}\right) \cdot \Delta x = \sum_{k=1}^{n} \left(\frac{16k^2}{n^2} + \frac{16k}{n} + 2\right) \cdot \frac{4}{n}$

Multiplicamos

$\sum_{k=1}^{n} \left(\frac{64k^2}{n^3} + \frac{64k}{n^2} + \frac{8}{n}\right)$

Separamos la suma

$\frac{64}{n^3} \sum_{k=1}^{n} k^2 + \frac{64}{n^2} \sum_{k=1}^{n} k + \sum_{k=1}^{n} \frac{8}{n}$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} \frac{8}{n} = \frac{8}{n} \cdot n = 8$$

Entonces:

$$\frac{64}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{64}{n^2} \cdot \frac{n(n+1)}{2} + 8$$

Simplificamos cada termino

$$\frac{64n(n+1)(2n+1)}{6n^3}$$

$$\frac{64n(n+1)}{2n^2} = \frac{32(n+1)}{n}$$

8

Calculamos el límite cuando $n \to \infty$

$$\lim_{n\to\infty} \frac{64n(n+1)(2n+1)}{6n^3} = \frac{64 \cdot 1 \cdot 2}{6} = \frac{128}{6} = \frac{64}{3}$$

$$\lim_{n\to\infty} \frac{32(n+1)}{n} = \lim_{n\to\infty} 32\left(1+\frac{1}{n}\right) = 32$$

$$\lim_{n\to\infty} 8 = 8$$

Sumamos todo

$$\frac{64}{3} + \underbrace{32+8} = \frac{64}{3} + \frac{40}{1} = \frac{120+64}{3} = \frac{184}{3}$$

Resultado final:

$$\int_0^4 (x^2+4x+2)\,dx = \frac{184}{3}$$