

$$1. \lim_{x \rightarrow \infty} \frac{3x^2 + 1}{x^2 + 5} = \frac{\infty}{\infty}$$

$$\frac{6x}{2x} = \frac{\infty}{\infty} \quad \frac{6}{2} = \frac{3}{1} = 3$$

$$2. \lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{\sin(0)}{0} = \frac{0}{0} = 1$$

$$3. \lim_{x \rightarrow \infty} \frac{5x + 2}{x^2 + 1} = \frac{\frac{5x}{x^2} + \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}} = \frac{\frac{5}{x} + \frac{2}{x^2}}{1 + \frac{1}{x^2}}$$

$$= \frac{0 + 0}{1 + 0} = \frac{0}{1} = 0$$

$$4. \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \frac{3^2 - 9}{3 - 3} = \frac{0}{0}$$

$$\frac{(x+3)(x-3)}{x-3} \quad 3+3 = 6$$

$$5. \lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2} = \frac{0}{0}$$

$$\frac{e^x - 1}{2x} = \frac{0}{0} = \frac{e^x}{2} = \frac{1}{2}$$

$$6. \lim_{x \rightarrow \infty} \frac{4x^3 - x}{2x^3 + 5} = \frac{\infty}{\infty}$$

$$\frac{12x^2 - 1}{6x^2} = \frac{24x}{12x} = \frac{24}{12} = \frac{12}{6} = \frac{6}{3} = \frac{2}{1} = 2$$

$$7. \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \frac{2^2 - 4}{2 - 2} = \frac{0}{0}$$

$$\frac{(x-2)(x+2)}{x-2} = 2+2 = 4$$

$$8. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{0}{0} \quad \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\frac{(1 - \cos x)(1 + \cos x)}{x^2(1 + \cos x)} = \frac{1 - \cos^2 x}{x^2(1 + \cos x)}$$

$$\sin^2 x + \cos^2 x = 1 = \frac{\sin^2 x}{x^2(1 + \cos x)}$$

$$= \frac{\sin x}{x} \cdot \frac{\sin x}{x} \cdot \frac{1}{1 + \cos x}$$

$$1 \cdot 1 \cdot \frac{1}{1 + \cos x} = 1 \cdot 1 \cdot \frac{1}{1 + 1} = 1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2}$$

$$9. \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \frac{0}{0} \quad \frac{(x-1)(x^2 + x(1) + 1^2)}{x-1}$$

$$\frac{(x-1)(x^2 + x + 1)}{x-1} = x^2 + x + 1 = 1^2 + 1 + 1 = 3$$

$$10. \lim_{x \rightarrow \infty} \frac{x^2 + x}{x^2} = \frac{\infty}{\infty} = \frac{2x + 1}{2x} = \frac{2}{2} = 1$$