

Francisco Javier Gómez Hernández

$$1: \int \frac{dx}{16x^2+4}$$

Identificando elementos y derivar U

$$U^2 = 16x^2 \quad a^2 = 4$$

$$U = 4x \quad a = 2$$

$$du = 4dx$$

$$= \frac{1}{4} \int \frac{4dx}{16x^2+4}$$

Integrar

$$= \frac{1}{4} \int \frac{du}{u^2+a^2} = \frac{1}{4} \left( \frac{1}{a} \arctan \frac{u}{a} \right) + C = \frac{1}{4} \left( \frac{1}{2} \arctan \frac{4x}{2} \right) + C$$

$$\text{Resultado} = \frac{1}{8} \arctan(2x) + C$$

$$2: \int \frac{dx}{\sqrt{25x^2+1}}$$

Identificando elementos y derivar U

$$U^2 = 25x^2 \quad a^2 = 1$$

$$U = 5x \quad a = 1 \quad = \frac{1}{5} \int \frac{5dx}{\sqrt{25x^2+1}}$$

$$du = 5dx$$

Integrar

$$= \frac{1}{5} \int \frac{du}{\sqrt{u^2+a^2}} = \frac{1}{5} \ln(u + \sqrt{u^2+a^2}) + C = \frac{1}{5} \ln(5x + \sqrt{25x^2+1}) + C$$

Resultado:

$$3: \int \frac{dx}{36-x^2}$$

Identificando elementos derivar U

$$U^2 = x^2 \quad a^2 = 36$$

$$U = x \quad du = dx \quad a = 6$$

Integrar

$$= \int \frac{du}{a^2-u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$$

Resultado:

$$= \frac{1}{2} \ln \left| \frac{x+6}{x-6} \right| + C$$



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$$4. \int \frac{dx}{\sqrt{4-4x^2}}$$

Identificar elementos y derivar u

$$U^2 = 4x^2 \quad a^2 = 4$$

$$U = 2x \quad a = 2$$

$$du = 2dx$$

$$= \frac{1}{2} \int \frac{2dx}{\sqrt{4-4x^2}}$$

Integrar

$$= \frac{1}{2} \int \frac{du}{\sqrt{a^2-u^2}} = \frac{1}{2} \arcsen \frac{u}{a} + c = \frac{1}{2} \arcsen \left( \frac{2x}{2} \right) + c$$

$$\text{Resultado} = \frac{1}{2} \arcsen(x) + c$$

$$5. \int \frac{dx}{2x\sqrt{4x^2-16}}$$

Identificar elementos y derivar u

$$U^2 = 4x^2 \quad U = 2x \quad a^2 = 16$$

$$du = 2xdx \quad a = 4$$

$$= \frac{1}{2} \int \frac{du}{2x\sqrt{4x^2-16}}$$

Integrar

$$= \frac{1}{2} \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{2} \left( \frac{1}{a} \operatorname{arcsec} \frac{u}{a} \right) + c = \frac{1}{2} \frac{1}{4} \operatorname{arcsec} \frac{2x}{4} + c$$

$$\text{Resultado} = \frac{1}{8} \operatorname{arcsec} \left( \frac{x}{2} \right) + c$$



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6.  $\int \sqrt{25-25x^2} dx$

Identificar elementos y derivar U

$$U^2 = 25x^2 \quad a^2 = 25$$

$$U = 5x \quad a = 5$$

$$dU = 5 dx$$

$$= \frac{1}{5} \int \sqrt{25-25x^2} 5 dx$$

Integrar

$$= \frac{1}{5} \int \sqrt{a^2 - U^2} dU = \frac{1}{5} \left( \frac{U}{2} \sqrt{a^2 - U^2} + \frac{a^2}{2} \arcsen \frac{U}{a} \right) + C$$

$$= \frac{1}{5} \left( 5x \frac{1}{2} \sqrt{25-25x^2} + \frac{25}{2} \arcsen \frac{5x}{5} \right) + C$$

$$\text{Resultado} = \frac{x}{2} \sqrt{25-25x^2} + \frac{5}{2} \arcsen(x) + C$$

7.  $\int \sqrt{x^2-49} dx$

Identificar elementos y derivar U

$$U^2 = x^2 \quad a^2 = 49$$

$$U = x \quad dU = dx \quad a = 7$$

- Integrar

$$= \int \sqrt{U^2 - a^2} dU = \frac{U}{2} \sqrt{U^2 - a^2} + \frac{a^2}{2} \ln(U + \sqrt{U^2 - a^2}) + C$$

$$\text{Resultado} = \frac{x}{2} \sqrt{x^2-49} + \frac{49}{2} \ln(x + \sqrt{x^2-49}) + C$$

8.  $\int \frac{dx}{4x^2-25}$

Identificar elementos y derivar U

$$U^2 = 4x^2 \quad dU = 2 dx \quad a^2 = 25$$

$$U = 2x \quad a = 5$$

$$= \frac{1}{2} \int \frac{2 dx}{4x^2-25}$$

- Integrar

$$\frac{1}{2} \int \frac{dU}{U^2 - a^2} = \frac{1}{2} \frac{1}{2a} \ln \left| \frac{U-a}{U+a} \right| + C = \frac{1}{2} \frac{1}{2(5)} \ln \left| \frac{2x-5}{2x+5} \right| + C$$

$$\text{Resultado} = \frac{1}{20} \ln \left| \frac{2x-5}{2x+5} \right| + C$$



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$$9: \int \frac{dx}{\sqrt{36x^2-1}}$$

Elementos de la integral y derivar U

$$U^2 = 36x^2 \quad dU = 6dx \quad a^2 = 1$$

$$U = 6x \quad a = 1$$

$$= \frac{1}{6} \int \frac{6dx}{\sqrt{36x^2-1}}$$

Integral

$$\frac{1}{6} \int \frac{dU}{\sqrt{U^2-a^2}} = \frac{1}{6} \ln(U + \sqrt{U^2-a^2}) + C$$

$$\text{Resultado} = \frac{1}{6} \ln(6x + \sqrt{36x^2-1}) + C$$

$$10: \int \frac{dx}{1-36x^2}$$

Identificar elementos y derivar U

$$U^2 = 36x^2 \quad dU = 6dx \quad a^2 = 1$$

$$U = 6x \quad a = 1$$

$$= \frac{1}{6} \int \frac{6dx}{1-36x^2}$$

$$\text{Integral} = \frac{1}{6} \int \frac{dU}{a^2-U^2} = \frac{1}{6} \frac{1}{2a} \ln \left| \frac{U+a}{U-a} \right| + C = \frac{1}{6} \frac{1}{2(1)} \ln \left| \frac{6x+1}{6x-1} \right| + C$$

$$\text{Resultado} = \frac{1}{12} \ln \left| \frac{6x+1}{6x-1} \right| + C$$

$$11: \int \frac{dx}{\sqrt{49x^2-4}}$$

Identificar elementos y derivar U

$$U^2 = 49x^2 \quad dU = 7dx \quad a^2 = 4$$

$$U = 7x \quad a = 2$$

Integral

$$\frac{1}{7} \int \frac{dU}{\sqrt{U^2-a^2}} = \frac{1}{7} \ln(U + \sqrt{U^2-a^2}) + C$$

$$\text{Resultado} = \frac{1}{7} \ln(7x + \sqrt{49x^2-4}) + C$$



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$$12. \int \frac{dx}{4x^2-1}$$

Identificar elementos y derivar U

$$U^2 = 4x^2 \quad du = 2dx \quad a^2 = 1$$

$$U = 2x \quad a = 1$$

$$= \frac{1}{2} \int \frac{2dx}{4x^2-1}$$

Integrar

$$= \frac{1}{2} \int \frac{du}{u^2-a^2} = \frac{1}{2} \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C = \frac{1}{2} \frac{1}{2(1)} \ln \left| \frac{2x-1}{2x+1} \right| + C$$

$$\text{Resultado} = \frac{1}{4} \ln \left| \frac{2x-1}{2x+1} \right| + C$$

$$13. \int \sqrt{1-9x^2} dx$$

Identificar elementos

$$U^2 = 9x^2 \quad du = 3dx \quad a^2 = 1$$

$$U = 3x \quad a = 1$$

$$= \frac{1}{3} \int \sqrt{1-9x^2} \cdot 3dx$$

Integrar

$$= \frac{1}{3} \int \sqrt{a^2-u^2} du = \frac{1}{3} \left( \frac{u}{2} \sqrt{a^2-u^2} + \frac{a^2}{2} \arcsen \frac{u}{a} \right) + C$$

$$\text{Resultado} = \frac{1}{2} \sqrt{1-9x^2} + \frac{1}{6} \arcsen(3x) + C$$

$$14. \int \frac{dx}{\sqrt{4x^2-9}}$$

Identificar elementos

$$U^2 = 4x^2 \quad du = 2dx \quad a^2 = 9$$

$$U = 2x \quad a = 3$$

$$= \frac{1}{2} \int \frac{2dx}{\sqrt{4x^2-9}}$$

Integrar

$$= \frac{1}{2} \int \frac{du}{\sqrt{u^2-a^2}} = \frac{1}{2} \ln(u + \sqrt{u^2-a^2}) + C$$

$$\text{Resultado} = \frac{1}{2} \ln(2x + \sqrt{4x^2-9}) + C$$

Scribe



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$$15 \int \frac{dx}{16x^2 - 25}$$

Identificar elementos

$$U^2 = 16x^2 \quad a^2 = 25$$

$$U = 4x \quad a = 5$$

$$du = 4dx$$

$$= \frac{1}{4} \int \frac{4dx}{16x^2 - 25}$$

Integrar

$$= \frac{1}{4} \int \frac{du}{U^2 - a^2} = \frac{1}{4} \frac{1}{2a} \ln \left| \frac{U-a}{U+a} \right| + C = \frac{1}{4} \frac{1}{2(5)} \ln \left| \frac{4x-5}{4x+5} \right| + C$$

$$\text{Resultado} = \frac{1}{40} \ln \left| \frac{4x-5}{4x+5} \right| + C$$

$$16: \int \frac{dx}{4x\sqrt{16x^2-1}}$$

Identificar elementos

$$U^2 = 16x^2 \quad a^2 = 1$$

$$U = 4x \quad a = 1$$

$$du = 4dx$$

$$= \frac{1}{4} \int \frac{4dx}{4x\sqrt{16x^2-1}}$$

Integrar

$$= \frac{1}{4} \int \frac{du}{U\sqrt{U^2-1}} = \frac{1}{4} - \frac{1}{a} \operatorname{arc} \operatorname{Sec} \frac{U}{a} + C$$

$$\text{Resultado} = \frac{1}{4} \operatorname{arc} \operatorname{Sec} (4x) + C$$