Metematicas

Nombre del docente: ojeda

Presenta: examen

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Semestre: 6

Técnico: enfermería

escolarizado

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① $\int (2x^2 - 5x + 3)^3 \, dx = \int (8x^6 - 60x^5 + 186x^4 - 305x^3 + 279x^2 - 135x + 27) \, dx$

$= 8\int x^6 dx - 60\int x^5 dx + 186\int x^4 dx + 305\int x^3 dx + 279\int x^2 dx - 135\int x\,dx + 27\int dx$

$= \frac{8x^7}{7} - \frac{60x^6}{6} + \frac{186x^5}{5} - 305\frac{x^4}{4} + \frac{279x^3}{3} - \frac{135x^2}{2} + 27x + C$

$= \frac{8}{7}x^7 - 10x^6 + \frac{186}{5}x^5 - \frac{305}{4}x^4 + 93x^3 - \frac{135}{2}x^2 + 27x + C$

② $\int \left(\frac{x^3 + 5x^2 - 4}{x^2}\right) dx = \int \frac{x^3 dx}{x^2} + 5\int \frac{x^2 dx}{x^2} - 4\int \frac{dx}{x^2} = \int x\,dx + 5\int dx - \frac{4dx}{x^2}$

$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$

③ $\int \frac{x^2}{4\sqrt{x^3+2}} dx = \int \frac{x^2}{4x^{3/2}+2} dx = \frac{1}{2}\int \frac{x^2}{2x^{3/2}+1} dx$

Si $u = 2x^{3/2} + 1 \quad du = 3\sqrt{x} \quad dx = \frac{1}{3\sqrt{x}} du$

$= \frac{1}{6}\int \frac{u-1}{u} du = \int \left(1 - \frac{1}{u}\right) du$

$= \int du - \int \frac{1}{u} du$

$= \int 2x^{3/2} + 1 - \int \frac{du}{2x^{3/2}+1}$

$= \frac{1}{2}\left(\frac{2x^{3/2}+1}{6}\right) \ln\left(\frac{2x^{3/2}+1}{6}\right)$

$= \frac{2x^{3/2}+1}{12} - \ln\frac{(2x^{3/2}+1)}{12} + C$

④ $\int 3\sqrt{(1-x^2)}\,x\,dx =$

$u = 1-x^2 \quad \frac{du}{dx} = -2x \quad dx = \frac{-1}{2x}\,du$

$= -\frac{3}{2}\int \sqrt{u}\,du$

$= \frac{-3}{2}\int u^{1/2}\,du = -\frac{3}{2}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= -\frac{3}{2}\,\frac{(1-x^2)^{\frac{3}{2}}}{\frac{3}{2}} + C = -(1+x^2)^{3/2} + C$

⑤ $\int \frac{(1+x)^2}{\sqrt{x}}\,dx = \int \left(x^{3/2} + 2\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$

$= \int x^{3/2}\,dx + 2\int \sqrt{x}\,dx + \int \frac{1}{\sqrt{x}}\,dx$

$= \frac{2x^{\frac{5}{2}}}{5} + 2\left(\frac{2x^{3/2}}{3}\right) + 2\sqrt{x} + C$

$= \frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + 2\sqrt{x} + C$

⑥ $\int \frac{\sqrt{x}}{x^2}\,dx = \int (x^{1/2})(x^{-2})\,dx = \int \frac{-3}{x^2}\,dx = \int \frac{dx}{x^{\frac{3}{2}}}$

$= \frac{-x^{\frac{1}{2}}}{\frac{1}{2}} + C = -(2)\frac{1}{x^{\frac{1}{2}}} + C = -\frac{1}{\sqrt{x}} + C$

⑦ $\int \frac{2x^3}{3\sqrt{x^2}}\,dx = \int \frac{2}{3}\,\frac{x^3}{x}\,dx = \frac{2}{3}\int x^2\,dx$

$= \frac{2}{3}\,\frac{x^3}{3} + C = \frac{2}{4}x^3 + C$

6. $\int \frac{dx}{4x^2+9} =$

$u = \frac{2x}{3} \quad \frac{du}{dx} = \frac{2}{3} \quad dx = \frac{3}{2}\, du$

$= \int \frac{3}{2(9u^2+9)} = du$

$= \frac{1}{6} \int \frac{1}{u^2+1}\, du$

$= \frac{\text{arcotamg}\left(\frac{3x}{3}\right)}{6} + C$

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⑨ $\int \frac{dx}{x^2+10+30} = \int \frac{1}{(x+5)^2+5} dx$

$u = \frac{x+5}{\sqrt{5}}$ $\frac{du}{dx} = \frac{1}{\sqrt{5}}$ $dx = \sqrt{5}\, du$

$= \int \frac{\sqrt{5}}{5u^2+5} = \frac{1}{5} \int \frac{1}{u^2+1} du$

$= \frac{1}{5} \text{arcotang}(u)$

$= \frac{\text{arcotang}\ \frac{x+5}{\sqrt{5}}}{\sqrt{5}} + C$

⑩ $\int \frac{1}{9x^2-16} dx = \frac{\ln(|3x-4|)}{24} \ \frac{\ln(|3x+4|)}{24} + C$

$\underbrace{9x^2-16}_{(3x-4)(3x+4)}$ $= \int \frac{1}{(3x-4)(3x+4)} = \frac{1}{8} \int \left(\frac{1}{8(3x-4)} - \frac{1}{8(3x+4)} \right) dx = \frac{1}{8}$

$= \frac{1}{8} \int \frac{1}{3-4} dx - \frac{1}{8} \int \frac{1}{3x+4} - \frac{1}{8} \left[\frac{\ln 3x-4}{3} - \frac{\ln 3x+4}{3} \right] =$

$u = 3x+4$ $\frac{du}{dx} = 3$ $dx = \frac{1}{3} du$

Luis Escandón