

2020

EXAMEN CALCULO



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INSTRUCCIONES: Resuelve de forma clara y correcta las siguientes derivadas, aplicando el método general (método de los cuatro pasos)

NOTA: LOS NUMEROS DESPUES DE LAS VARIABLES Y LOS PARENTESIS SON EXPONENTES

- 1.- $Y = 2X^3 - 3X + 9$
- 2.- $Y = 4 / X^2$
- 3.- $Y = 5 / 4 + X^2$
- 4.- $Y = X + 2 / X$
- 5.- $Y = (a - bx)^2$
- 6.- $Y = 2 / X^2 + 4$
- 7.- $Y = (1 + 2X)^2$
- 8.- $Y = 2 - X / X - 2$

1.- $Y = 2X^3 - 3X + 9$

$$Y + \Delta Y = 2(x + \Delta x)^3 - 3(x + \Delta x) + 9$$

$$1(x + \Delta x)^3 = (x + \Delta x)(x + \Delta x)(x + \Delta x)$$

$$= x^3 + x^2\Delta x + 2x^2\Delta x + 2x\Delta x^2 + x\Delta x^2 + \Delta x^3$$

$$= x^3 + 3x^2\Delta x + 3x\Delta x^2 + \Delta x^3$$

$$= 2x^3 + 6x^2\Delta x + 6x\Delta x^2 + 2\Delta x^3 - 3x - 3\Delta x + 9$$

$$-X = 6x^2\Delta x + 6x\Delta x^2 + 2\Delta x^3 - 3\Delta x$$

$$\Delta Y = 6x^2\Delta x + 6x\Delta x^2 + 2\Delta x^3 - 3\Delta x$$

$$\Delta Y = \Delta x (6x^2 + 6x\Delta x + 2\Delta x^2 - 3)$$

$$\frac{\Delta Y}{\Delta x} = 6x^2 + 6x\Delta x + 2\Delta x^2 - 3$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta Y}{\Delta x} = \lim_{\Delta x \rightarrow 0} (6x^2 + 6x\Delta x + 2\Delta x^2 - 3)$$

$$Y' = 6x^2 + 6x(0) + 2(0)^2 - 3$$

$$Y' = 6x^2 - 3$$

2.- $Y = \frac{4}{x^2}$

$$Y + \Delta Y = \frac{4}{(x + \Delta x)^2} = \frac{4}{x^2 + 2x\Delta x + \Delta x^2}$$

$$\Delta Y = \frac{4}{x^2 + 2x\Delta x + \Delta x^2} - \frac{4}{x^2}$$

$$\Delta Y = \frac{4}{x^2 + 2x\Delta x + \Delta x^2} - \frac{4}{x^2}$$

$$\Delta Y = \frac{4(x^2) - 4(x^2 + 2x\Delta x + \Delta x^2)}{(x^2 + 2x\Delta x + \Delta x^2)(x^2)}$$

$$\Delta Y = \frac{-8x\Delta x - 4\Delta x^2}{(x^2 + 2x\Delta x + \Delta x^2)(x^2)}$$

$$\Delta Y = \frac{-8x\Delta x - 4\Delta x^2}{\Delta x (x^2 + 2x\Delta x + \Delta x^2)(x^2)}$$

$$\Delta Y = \frac{-8x - 4\Delta x}{(x^2 + 2x\Delta x + \Delta x^2)(x^2)}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta Y}{\Delta x} = \frac{-8x - 4\Delta x}{(x^2 + 2x(0) + (0)^2)(x^2)}$$

$$= \frac{-8x - 4(0)}{(x^2)(x^2)} = \frac{-8x}{(x^2)^2}$$

$$3. y = \frac{5}{4+x^2}$$

$$y + \Delta y = \frac{5}{4+(x+\Delta)^2} = \frac{5}{4+x^2+2x\Delta+\Delta^2}$$

$$\Delta y = \frac{5}{4+x^2+2x\Delta+\Delta^2} - y$$

$$\Delta y = \frac{5}{4+x^2+2x\Delta+\Delta^2} - \frac{5}{4+x^2}$$

$$\Delta y = \frac{20 + 5x^2 - 20 - 5x^2 - 10x\Delta - 5\Delta^2}{(4+x^2+2x\Delta+\Delta^2)(4+x^2)}$$

$$\Delta y = \frac{-10\Delta x - 5\Delta^2}{(4+x^2+2x\Delta+\Delta^2)(4+x^2)}$$

$$\frac{\Delta x}{1}$$

$$\Delta y = \frac{-10x\Delta - 5\Delta^2}{\Delta(4+x^2+2x\Delta+\Delta^2)(4+x^2)}$$

$$\frac{Dy}{Dx} = \frac{\Delta y (-10x - 5\Delta)}{\Delta(4+x^2+2x\Delta+\Delta^2)(4+x^2)}$$

$$\lim_{\Delta \rightarrow 0} \frac{Dy}{Dx} = \frac{-10x - 5(0)}{(4+2x(0)+0^2)(4+x^2)} = \frac{-10x}{(4+x^2)(4+x^2)}$$

$$= \frac{-10x}{(4+x^2)^2}$$

$$4. y = \frac{x+2}{x}$$

$$y + \Delta y = \frac{x+2}{x} = y' = \frac{d}{dx} \left(\frac{x+2}{x} \right)$$

$$y' = \frac{d}{dx} \left(\frac{x}{x} + \frac{2}{x} \right) = y' = \frac{d}{dx} \left(1 + \frac{2}{x} \right)$$

$$y' = \frac{d}{dx} (1) + \frac{d}{dx} \left(\frac{2}{x} \right)$$

$$y' = 0 - 2x^{-2}$$

$$y' = -\frac{2}{x^2}$$

$$5. y = (a - bx)^2$$

$$y = (a - bx)(a - bx)$$

$$y = a^2 - 2abx + b^2x^2$$

$$y = a^2 - 2ab(x + \Delta x) + b^2(x + \Delta x)^2$$

$$= (x + \Delta x)(x + \Delta x) = x^2 + x\Delta x + x\Delta x + \Delta x^2$$

$$= x^2 + 2x\Delta x + \Delta x^2$$

$$= a^2 - 2ab(x + \Delta x) + b^2(x + \Delta x)^2$$

$$= a^2 - 2ab(x^2 - 2x\Delta x + \Delta x^2) + b^2x^2 - b^2\Delta x^2$$

$$= a^2 - 2abx^2 + 4abx\Delta x + 2ab\Delta x^2 - b^2\Delta x^2$$

$$- (a^2 - 2abx + b^2x^2)$$

$$= a^2 - 2abx^2 + 4abx\Delta x + 2ab\Delta x^2 - b^2\Delta x^2 - a^2 + 2abx - b^2x^2$$

$$= \frac{4abx\Delta x - b\Delta x^2 - 2a}{\Delta x} = 4abx + b(1)x - 2ab$$

$$\Delta x \rightarrow 0 = 6ab - 2ab$$

$$6. y = \frac{2}{x^2+4}$$

$$y + \Delta y = \frac{2}{(x + \Delta x)^2 + 4} = \frac{2}{x^2 + 2x\Delta x + \Delta x^2 + 4}$$

$$\Delta y = \frac{2}{x^2 + 2x\Delta x + \Delta x^2 + 4} - y$$

$$\Delta y = \frac{2}{x^2 + 2x\Delta x + \Delta x^2 + 4} - \frac{2}{x^2 + 4}$$

$$\Delta y = \frac{2(x^2 + 4) - 2(x^2 + 2x\Delta x + \Delta x^2 + 4)}{(x^2 + 2x\Delta x + \Delta x^2 + 4)(x^2 + 4)}$$

$$\Delta y = \frac{-2x^2 + 8 - 2x^2 - 4x\Delta x - 2\Delta x^2 - 8}{(x^2 + 2x\Delta x + \Delta x^2 + 4)(x^2 + 4)}$$

$$\frac{\Delta y}{\Delta x} = \frac{-4x\Delta x - 2\Delta x^2}{(x^2 + 2x\Delta x + \Delta x^2 + 4)(x^2 + 4)\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{\Delta x(-4x - 2\Delta x)}{(x^2 + 2x\Delta x + \Delta x^2 + 4)(x^2 + 4)}$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta x} = \frac{-4x - 2(0)x}{(x^2 + 2x(0) + (0)^2 + 4)(x^2 + 4)}$$

$$= \frac{-4x}{(x^2 + 4)(x^2 + 4)} = \frac{-4x}{(x^2 + 4)^2}$$