

$$\begin{aligned}
1. & \int x^2 \operatorname{sen} x \, dx \\
&= x^2 (-\cos(x)) - \int -\cos(x) 2x \, dx \\
&= x^2 (-\cos(x)) - 1(-2) \int \cos(x) x \, dx \\
&= x^2 (-\cos(x)) + 2 \int x \cos(x) \, dx \\
&= x^2 (-\cos(x)) + 2 (x \operatorname{sen}(x) - \int \operatorname{sen}(x) \, dx) \\
&= x^2 (-\cos(x)) + 2 (x \operatorname{sen}(x) - (-\cos(x))) \\
&= -x^2 \cos x + 2x \operatorname{sen} x + 2 \cos x \\
&= -x^2 \cos x + 2x \operatorname{sen} x + 2 \cos x + C
\end{aligned}$$



$$2. \int x^3 e^{2x} dx$$

$$= x^3 \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} 3x^2 dx$$

$$= x^3 \frac{e^{2x}}{2} - \frac{1}{2} 3 \int e^{2x} x^2 dx$$

$$= x^3 \frac{e^{2x}}{2} - \frac{3}{2} \left( x^2 \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} 2x dx \right)$$

$$= x^3 \frac{e^{2x}}{2} - \frac{3}{2} \left( x^2 \frac{e^{2x}}{2} - \int e^{2x} x dx \right)$$

$$= x^3 \frac{e^{2x}}{2} - \frac{3}{2} \left( x^2 \frac{e^{2x}}{2} - \int x e^{2x} dx \right)$$

$$= x^3 \frac{e^{2x}}{2} - \frac{3}{2} \left( x^2 \frac{e^{2x}}{2} - \left( x \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx \right) \right)$$

$$= x^3 \frac{e^{2x}}{2} - \frac{3}{2} \left( x^2 \frac{e^{2x}}{2} - \left( x \frac{e^{2x}}{2} - \frac{1}{2} \int e^{2x} dx \right) \right)$$

$$= x^3 \frac{e^{2x}}{2} - \frac{3}{2} \left( x^2 \frac{e^{2x}}{2} - \left( x \frac{e^{2x}}{2} - \frac{1}{2} \left( \frac{1}{2} \right) e^{2x} \right) \right)$$

$$= \frac{x^3 e^{2x}}{2} - \frac{3x^2 e^{2x} - 3x e^{2x} - \frac{3e^{2x}}{8}}$$

$$= \frac{x^3 e^{2x}}{2} - \frac{3x^2 e^{2x} - 3x e^{2x}}{4} - \frac{3e^{2x}}{8} + C$$

$$3 \int x^2 \sqrt{1-x} dx$$

$$= \int -t^2 \sqrt{t} + 2t \sqrt{t} - \sqrt{t} dt$$

$$= \int -t^2 \left(t \frac{1}{2}\right) + 2t \left(t \frac{1}{2}\right) - t \frac{1}{2} dt$$

$$= \int -t^2 \left(t \frac{1}{2}\right) + 2t \left(t \frac{1}{2}\right) - t \frac{1}{2} dt$$

$$= \int -t \frac{5}{5} + 2t \frac{3}{2} - t \frac{1}{2} dt$$

$$= - \int t \frac{5}{5} dt + \int 2t \frac{3}{2} dt - \int t \frac{1}{2} dt$$

$$= \frac{-2t^3 \sqrt{t}}{7} + \frac{4t^2 \sqrt{t}}{5} - \frac{2t \sqrt{t}}{3}$$

$$= \frac{-2(1-x)^3 (\sqrt{1-x})}{7} + \frac{4(1-x)^2 (\sqrt{1-x})}{5} - \frac{2(1-x) (\sqrt{1-x})}{3}$$

$$= \frac{-2\sqrt{1-x} (1-3x+3x^2-x^3)}{7} + \frac{4\sqrt{1-x} (1-2x+x^2)}{5} - \frac{2(1-x) (\sqrt{1-x})}{3}$$

$$= \frac{-2\sqrt{1-x} (1-3x+3x^2-x^3)}{7} + \frac{4\sqrt{1-x} (1-2x+x^2)}{5} - \frac{2(1-x) (\sqrt{1-x})}{3} + C$$

$$\begin{aligned}
 6 &= \int \sin 3x \cos 2x \, dx \\
 &= \int \frac{1}{2} (\sin 5x + \sin x) \, dx \\
 &= \frac{1}{2} \int \sin 5x + \sin x \, dx \\
 &= \frac{1}{2} \int \sin 5x \, dx + \int \sin x \, dx \\
 &= \frac{1}{2} \left( -\frac{\cos 5x}{5} - \frac{\cos x}{2} \right) \\
 &= -\frac{\cos 5x}{10} - \frac{\cos x}{2} \\
 &= \frac{\cos 5x}{10} - \frac{\cos x}{2} + c
 \end{aligned}$$

$$\begin{aligned}
 7 &= \int \sin(\ln x) \, dx \\
 &= \sin(t) e^t - \int e^t \cos(t) \, dt \\
 &= \sin(t) e^t - \int \cos(t) \, dt \\
 &= \sin(t) e^t - (\cos(t) e^t - \int e^t (-\sin(t)) \, dt) \\
 &= \sin(t) e^t - \cos(t) e^t + \int e^t (-\sin(t)) \, dt \\
 &= \int e^t \sin(t) \, dt = \sin(t) e^t - \cos(t) e^t - \int e^t \sin(t) \, dt \\
 &= \int e^t \sin(t) \, dt = \sin(t) e^t - \cos(t) e^t - \int e^t \sin(t) \, dt \\
 &= \int e^t \sin(t) \, dt + \int e^t \sin(t) \, dt = \sin(t) e^t - \cos(t) e^t \\
 &= 2 \int e^t \sin(t) \, dt = \sin(t) e^t - \cos(t) e^t \\
 &= \int e^t \sin(t) \, dt = \frac{\sin(t) e^t}{2} - \frac{\cos(t) e^t}{2} \\
 &= \int \frac{\sin(\ln(x)) e^{\ln(x)}}{2} - \frac{\cos(\ln(x)) e^{\ln(x)}}{2} \\
 &= \frac{\sin(\ln(x)) x}{2} - \frac{\cos(\ln(x)) x}{2} \\
 &= \frac{\sin(\ln(x)) x - \cos(\ln(x)) x}{2} + c
 \end{aligned}$$