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Materia: Matemática Aplicada

Trabajo: Examen

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1- $\int x^2 \,\text{sen}\, x \, dx$

$\int u\,dv = u.v - \int v\,du$

$u = x^2$ $\quad$ $dv = \text{sen}\, x \, dx$

$du = 2x.dx$ $\quad$ $\int dv = \int dv = \int \text{sen}\, x\, dx$

$du = 2x\,dx$ $\quad$ $v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$\int x^2 \,\text{sen}\, x\,dx = x^2(-\cos x) - \int (-\cos x)\, 2x\,dx$

$= -x^2 \cos x + \int 2x.\cos x\,dx$

$\int 2x \,.\, \cos dx$

$u = 2x$ $\quad$ $dv = \cos x\,dx$

$du = 2dx$ $\quad$ $v = \text{sen}\, x$

$\int u\,dv = uv - \int v\,du$

$\int 2x \cos x\,dx = 2x.\text{sen}\,x - \int (\text{sen}\, x)\, 2\,dx$

$= 2x \,.\, \text{sen}\,x - 2\int \text{sen}\, x\,dx$

$= 2x.\,\text{sen}\,x + 2\cos x + C$

$\int x^2 \,\text{sen}\, x\,dx = -x^2\cos x + \int 2x.\cos x\,dx$

$= -x^2 \cos x + 2x\,\text{sen}\,x + 2\cos x + C$

2) $\int x^3 e^{2x}\, dx$

Let $I = \int \underset{A}{x^3}\, \underset{E}{e^{2x}}\, dx$

$= x^3 \int e^{2x} dx - \int \left\{ \frac{d}{dx}(x^3) \cdot \int e^{2x} dx \right\} dx$

$= x^3 \times \frac{e^{2x}}{2} - \int 3x^2 \cdot \frac{e^{2x}}{2}\, dx$

$= \frac{x^3 e^{2x}}{2} - \frac{3}{2} \int x^2 \cdot e^{2x} dx$

$= \frac{x^3 e^{2x}}{2} - \frac{3}{2} \left[x^2 \int e^{2x} dx - \int \left\{ \frac{d}{dx}(x^2) \cdot \int e^{2x} dx \right\} dx \right]$

$= \frac{x^3 e^{2x}}{2} - \frac{3}{2} \left[x^2 \cdot \frac{e^{2x}}{2} - \int 2x \cdot \frac{e^{2x}}{2}\, dX \right]$

2.

$$= \frac{x^3e^{2x}}{2} - \frac{3}{4}x^2e^{2x} + \frac{3}{2}\int xe^{2x}\,dx$$

$$= \frac{x^3e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \frac{3}{2}\left[x\int e^{2x}dx - \int\left\{\frac{d}{dx}(x).\int e^{2x}dx\right\}dx\right]$$

$$= \frac{x^3e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \frac{3}{2}\left[x.\frac{e^{2x}}{2} - \int\frac{e^{2x}}{2}dx\right]$$

$$= \frac{x^2e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \frac{3}{2}\left[\frac{xe^{2x}}{2} - \frac{1}{2}\times\frac{e^{2x}}{2}\right] + C$$

$$= \frac{x^2e^{2x}}{2} - \frac{3x^2e^{2x}}{4} + \frac{3xe^{2x}}{4} - \frac{3}{8}e^{2x} + C$$

$$= \frac{e^{2x}}{2}\left[x^2 - \frac{3x^2}{2} + \frac{3x}{2} - \frac{3}{4}\right] + C$$

3.- $\int x2 \sqrt{(1-x)}\, dx$

$$= \int (1-v^2)^2 v(-2v)\, dv = 2\int (1-2v^2+v^4)(-v^2)\, dv$$

$$= \int (-v^2+2v^4-v^6)\, dv$$

$$= -2\int v^2 dv + 4\int v^4 dv - 2\int v^6 dv$$

$$= -2\frac{v^3}{3} + 4\frac{v^5}{5} - 2\frac{v^7}{7} + C$$

$$= -\frac{2}{3}\left(\sqrt{1-x}\right)^3 + \frac{4}{5}\left(\sqrt{1-x}\right)^5 - \frac{2}{7}\left(\sqrt{1-x}\right)^7 + C$$

4.- $\int e^{ax} \cos bx\, dx$

$= \cos bx \left(\frac{1}{a} e^{ax}\right) - \int \frac{1}{a} e^{ax} (-b \operatorname{sen} bx)\, dx$

$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{a} \int e^{ax} \operatorname{sen} bx\, dx$

$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{a} \left(\frac{1}{a} e^{ax} \operatorname{sen} bx - \int \frac{1}{a} e^{ax} (b \cos bx)\, dx\right)$

$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{a} \left(\frac{1}{a} e^{ax} \operatorname{sen} bx - \frac{b}{a} \int e^{ax} \cos bx\, dx\right)$

$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{a^2} e^{ax} \operatorname{sen} bx - \frac{b^2}{a^2} \int e^{ax} \cos bx\, dx$

$\int e^{ax} \cos bx\, dx = \frac{1}{a} e^{ax} \cos bx + \frac{b}{a^2} e^{ax} \operatorname{sen} bx - \frac{b^2}{a^2} \int e^{ax} \cos bx\, dx$

$\int e^{ax} \cos bx\, dx + \frac{b^2}{a^2} \int e^{ax} \cos bx\, dx = \frac{1}{a} e^{ax} \cos bx + \frac{b}{a^2} e^{ax} \operatorname{sen} bx$

$\frac{a^2+b^2}{a^2} \int e^{ax} \cos bx\, dx = \frac{1}{a} e^{ax} \cos bx + \frac{b}{a^2} e^{ax} \operatorname{sen} bx$

4.

$$\int e^{ax} \cos bx \, dx = \frac{a^2}{a^2+b^2}\left(\frac{1}{a} e^{ax} \cos bx + \frac{b}{a^2} e^{ax} \operatorname{sen} bx\right)$$

$$\int e^{ax} \cos bx \, dx = \frac{a}{a^2+b^2} e^{ax} \cos bx + \frac{b}{a^2+b^2} e^{ax} \operatorname{sen} bx + C$$

5. $\operatorname{Sen}^3 x \, dx$

$= \int \operatorname{sen}^2 x \cdot \operatorname{Sen} x \, dx$

$= \int (1 - \cos^2 x) \operatorname{Sen} x \, dx = \int (\operatorname{sen} x - \cos^2 x \operatorname{sen} x) \, dx$

$= \int \operatorname{sen} x \, dx - \int \cos^2 x \operatorname{sen} x \, dx$

$= -\cos x + \int \cos^2 x (-\operatorname{sen} x) \, dx$

$= -\cos(x) + \frac{\cos^3 x}{3} + C$

6. $\int \operatorname{sen} 3x \cos 2x \, dx$

$= \int \frac{1}{2} (\operatorname{sen} 5x + \operatorname{sen} x) \, dx$

$= \frac{1}{2} \int \operatorname{sen} 5x + \operatorname{sen} x \, dx$

$= \frac{1}{2} \int \operatorname{sen} 5x \, dx + \int \operatorname{sen} x \, dx$

$= \frac{1}{2} \left(-\frac{\cos 5x}{5} - \cos x \right.$

$= -\frac{\cos 5x}{10} - \frac{\cos x}{2}$

$= -\frac{\cos 5x}{10} - \frac{\cos x}{2} + C$

7. $\int \text{sen}(\ln x)\,dx = x\,\text{sen}(\ln x) - x\cos(\ln x) - \int \text{sen}(\ln x)\,dx$

$$\int \text{sen}(\ln x)\,dx + \int \text{sen}(\ln x)\,dx = x\,\text{sen}(\ln x) - x\cos(\ln x)$$

$$2\int \text{sen}(\ln x)\,dx = x\,\text{sen}(\ln x) - x\cos(\ln x)$$

$$\int \text{sen}(\ln x)\,dx = \frac{x\,\text{sen}(\ln x) - x\cos(\ln x)}{2} + C$$

$$\int \text{sen}(\ln x)\,dx = \frac{1}{2}x\,\text{sen}(\ln x) - \frac{1}{2}x\cos(\ln x) + C$$

8. $x^2 \ln x \, dx$

$= \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} \, dx$

$= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 \, dx$

$= \frac{x^3}{3} \ln x - \frac{1}{3} \frac{x^3}{3} + C$

$= \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$

10. $\int \ln(x^2)\cos(x)\,dx$

$= \ln(x^2)\,\mathrm{sen}(x) - \int \mathrm{sen}(x)\, 2\,\frac{1}{x}\,dx$

$= \ln(x^2)\,\mathrm{sen}(x) - 2\int \mathrm{sen}(x)\,\frac{1}{x}\,dx$

$= \ln(x^2)\,\mathrm{sen}(x) - 2\int \frac{\mathrm{sen}(x)}{x}\,dx$

$= \ln(x^2)\,\mathrm{sen}(x)$