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MATERIA:MATEMATICAS APLICADA

# Capítulo 31

## Integración de funciones hiperbólicas

**FORMULAS DE INTEGRACION**

$$\int \operatorname{senh} u\,du = \cosh u + C \qquad \int \operatorname{sech}^2 u\,du = \operatorname{tagh} u + C$$

$$\int \cosh u\,du = \operatorname{senh} u + C \qquad \int \operatorname{csch}^2 u\,du = -\coth u + C$$

$$\int \operatorname{tagh} u\,du = \ln \cosh u + C \qquad \int \operatorname{sech} u \operatorname{tagh} u\,du = -\operatorname{sech} u + C$$

$$\int \coth u\,du = \ln|\operatorname{senh} u| + C \qquad \int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$$

$$\int \frac{du}{\sqrt{u^2+a^2}} = \operatorname{senh}^{-1}\frac{u}{a} + C \qquad \int \frac{du}{a^2-u^2} = \frac{1}{a}\operatorname{tagh}^{-1}\frac{u}{a} + C, \quad u^2 < a^2$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\frac{u}{a} + C, \quad u > a > 0 \qquad \int \frac{du}{u^2-a^2} = -\frac{1}{a}\coth^{-1}\frac{u}{a} + C, \quad u^2 > a^2$$

## Problemas resueltos

1. $\int \operatorname{senh} \tfrac{1}{2}x\,dx = 2\cosh \tfrac{1}{2}x + C$

2. $\int \cosh 2x\,dx = \tfrac{1}{2}\operatorname{senh} 2x + C$

3. $\int \operatorname{sech}^2 (2x-1)\,dx = \tfrac{1}{2}\operatorname{tagh}(2x-1) + C$

4. $\int \operatorname{csch} 3x \coth 3x\,dx = -\tfrac{1}{3}\operatorname{csch} 3x + C$

5. $\int \operatorname{sech} x\,dx = \int \frac{1}{\cosh x}dx = \int \frac{\cosh x}{\cosh^2 x} = \int \frac{\cosh x}{1+\operatorname{senh}^2 x}dx = \operatorname{arc\,tag}(\operatorname{senh} x) + C$

6. $\int \operatorname{senh}^2 x\,dx = \frac{1}{2}\int (\cosh 2x - 1)\,dx = \tfrac{1}{4}\operatorname{senh} 2x - \tfrac{1}{2}x + C$

7. $\int \operatorname{tagh}^2 2x\,dx = \int (1-\operatorname{sech}^2 2x)\,dx = x - \tfrac{1}{2}\operatorname{tagh} 2x + C$

8. $\int \cosh^3 \tfrac{1}{2}x\,dx = \int (1+\operatorname{senh}^2 \tfrac{1}{2}x)\cosh \tfrac{1}{2}x\,dx = 2\operatorname{senh}\tfrac{1}{2}x + \tfrac{2}{3}\operatorname{senh}^3 \tfrac{1}{2}x + C$

9. $\int \operatorname{sech}^4 x\,dx = \int (1-\operatorname{tagh}^2 x)\operatorname{sech}^2 x\,dx = \operatorname{tagh} x - \tfrac{1}{3}\operatorname{tagh}^3 x + C$

10. $\int e^x \cosh x\,dx = \int e^x\left(\frac{e^x+e^{-x}}{2}\right)dx = \frac{1}{2}\int (e^{2x}+1)\,dx = \frac{1}{4}e^{2x} + \frac{1}{2}x + C$

11. $\int x \operatorname{senh} x\,dx = \int x\left(\frac{e^x-e^{-x}}{2}\right)dx = \frac{1}{2}\int xe^x\,dx - \frac{1}{2}\int xe^{-x}\,dx$

$$= \frac{1}{2}(xe^x - e^x) - \frac{1}{2}(-xe^{-x} - e^{-x}) + C = x\left(\frac{e^x+e^{-x}}{2}\right) - \frac{e^x-e^{-x}}{2} + C$$

$$= x\cosh x - \operatorname{senh} x + C$$

**66)** $\int \frac{\sin^n cx\ dx}{\cos cx} = -\frac{\sin^{n-1} cx}{c(n-1)} + \int \frac{\sin^{n-2} cx\ dx}{\cos cx} \qquad (\text{for } n \neq 1)$

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**67)** $\int \frac{\sin^n cx\ dx}{\cos cx} = -\frac{\sin^{n-1} cx}{c(n-1)} + \int \frac{\sin^{n-2} cx\ dx}{\cos cx} \qquad (\text{for } n \neq 1)$

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**68)** $\int \frac{\sin^n cx\ dx}{\cos^m cx} = \frac{\sin^{n+1} cx}{c(m-1)\cos^{m-1} cx} - \frac{n-m+2}{m-1} \int \frac{\sin^n cx\ dx}{\cos^{m-2} cx} \qquad (\text{para } m \neq 1)$

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**69)** $\int \frac{\cos cx\ dx}{\sin^n cx} = -\frac{1}{c(n-1)\sin^{n-1} cx} \qquad (\text{para } n \neq 1)$

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**70)** $\int \frac{\cos^2 cx\ dx}{\sin cx} = \frac{1}{c}\left(\cos cx + \ln\left|\tan \frac{cx}{2}\right|\right)$

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**71)** $\int \frac{\cos^2 cx\ dx}{\sin^n cx} = -\frac{1}{n-1}\left(\frac{\cos cx}{c\sin^{n-1} cx)} + \int \frac{dx}{\sin^{n-2} cx}\right) \qquad (\text{para } n \neq 1)$

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**72)** $\int \frac{\cos^n cx\ dx}{\sin^m cx} = -\frac{\cos^{n+1} cx}{c(m-1)\sin^{m-1} cx} - \frac{n-m-2}{m-1} \int \frac{cos^n cx\ dx}{\sin^{m-2} cx} \qquad (\text{para } m \neq 1)$

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**73)** $\int \frac{\cos^n cx\ dx}{\sin^m cx} = \frac{\cos^{n-1} cx}{c(n-m)\sin^{m-1} cx} + \frac{n-1}{n-m} \int \frac{cos^{n-2} cx\ dx}{\sin^m cx} \qquad (\text{para } m \neq n)$

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**74)** $\int \frac{\cos^n cx\ dx}{\sin^m cx} = -\frac{\cos^{n-1} cx}{c(m-1)\sin^{m-1} cx} - \frac{n-1}{m-1} \int \frac{cos^{n-2} cx\ dx}{\sin^{m-2} cx} \qquad (\text{para } m \neq 1)$