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**TEMA: INTEGRALES DE FUNCIONES HIPERBOLICAS**

**GRADO: 6º A**

**FECHA: 25/06/2020**

BIBLIOGRAFIA: <https://www.derivadas.es/integrales-trigonometricas-e-hiperbolicas/#:~:text=Integrales%20de%20funciones%20hiperb%C3%B3licas,-Las%20funciones&text=Las%20funciones%20hiperb%C3%B3licas%20b%C3%A1sicas%20son,la%20tangente%20hiperb%C3%B3lica%20tanhx>

## INTEGRALES DE FUNCIONES HIPERBOLICAS

Las funciones hiperbólicas son análogas a las funciones ordinarias.

Las funciones hiperbólicas básicas son:

- El seno hiperbólico  $\sinh x$
- El coseno hiperbólico  $\cosh x$  de donde podemos derivar la tangente hiperbólica  $\tanh x$

Las identidades trigonométricas hiperbólicas formulario más básicas son las siguientes:

$$75) \int \sinh cx \, dx = \frac{1}{c} \cosh cx$$

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$$76) \int \cosh cx \, dx = \frac{1}{c} \sinh cx$$

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$$77) \int \sinh^2 cx \, dx = \frac{1}{4c} \sinh 2cx - \frac{x}{2}$$

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$$78) \int \cosh^2 cx \, dx = \frac{1}{4c} \sinh 2cx + \frac{x}{2}$$

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$$79) \int \sinh^n cx \, dx = \frac{1}{cn} \sinh^{n-1} cx \cosh cx - \frac{n-1}{n} \int \sinh^{n-2} cx \, dx \quad (\text{para } n > 0)$$

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$$80) \int \sinh^n cx \, dx = \frac{1}{c(n+1)} \sinh^{n+1} cx \cosh cx - \frac{n+2}{n+1} \int \sinh^{n+2} cx \, dx \quad (\text{para } n < 0, n \neq -1)$$

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$$81) \int \cosh^n cx \, dx = \frac{1}{cn} \sinh cx \cosh^{n-1} cx + \frac{n-1}{n} \int \cosh^{n-2} cx \, dx \quad (\text{para } n > 0)$$

$$82) \int \cosh^n cx \, dx = -\frac{1}{c(n+1)} \sinh cx \cosh^{n+1} cx + \frac{n+2}{n+1} \int \cosh^{n+2} cx \, dx \quad (\text{para } n < 0, n \neq -1)$$

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$$83) \int \cosh^n cx \, dx = -\frac{1}{c(n+1)} \sinh cx \cosh^{n+1} cx + \frac{n+2}{n+1} \int \cosh^{n+2} cx \, dx \quad (\text{para } n < 0, n \neq -1)$$

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$$84) \int \frac{dx}{\sinh cx} = \frac{1}{c} \ln \left| \tanh \frac{cx}{2} \right|$$

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$$85) \int \frac{dx}{\cosh cx} = \frac{2}{c} \arctan e^{cx}$$

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$$86) \int \frac{dx}{\sinh^n cx} = \frac{\cosh cx}{c(n-1) \sinh^{n-1} cx} - \frac{n-2}{n-1} \int \frac{dx}{\sinh^{n-2} cx} \quad (\text{para } n \neq 1)$$

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$$87) \int \frac{dx}{\cosh^n cx} = \frac{\sinh cx}{c(n-1) \cosh^{n-1} cx} + \frac{n-2}{n-1} \int \frac{dx}{\cosh^{n-2} cx} \quad (\text{para } n \neq 1)$$

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$$88) \int \frac{\cosh^n cx}{\sinh^m cx} dx = \frac{\cosh^{n-1} cx}{c(n-m) \sinh^{m-1} cx} + \frac{n-1}{n-m} \int \frac{\cosh^{n-2} cx}{\sinh^m cx} dx \quad (\text{para } m \neq n)$$

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$$89) \int \frac{\sinh^m cx}{\cosh^n cx} dx = \frac{\sinh^{m-1} cx}{c(m-n) \cosh^{n-1} cx} + \frac{m-1}{m-n} \int \frac{\sinh^{m-2} cx}{\cosh^n cx} dx \quad (\text{para } m \neq n)$$

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$$90) \int x \sinh cx dx = \frac{1}{c} x \cosh cx - \frac{1}{c^2} \sinh cx$$

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$$91) \int x \cosh cx dx = \frac{1}{c} x \sinh cx - \frac{1}{c^2} \cosh cx$$

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$$92) \int \tanh cx dx = \frac{1}{c} \ln |\cosh cx|$$

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$$93) \int \coth cx dx = \frac{1}{c} \ln |\sinh cx|$$

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$$94) \int \tanh^n cx dx = -\frac{1}{c(n-1)} \tanh^{n-1} cx + \int \tanh^{n-2} cx dx \quad (\text{para } n \neq 1)$$

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$$95) \int \coth^n cx dx = -\frac{1}{c(n-1)} \coth^{n-1} cx + \int \coth^{n-2} cx dx \quad (\text{para } n \neq 1)$$

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$$96) \int \sinh bx \sinh cx \, dx = \frac{1}{b^2 - c^2} (b \sinh cx \cosh bx - c \cosh cx \sinh bx) \quad (\text{para } b^2 \neq c^2)$$

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$$97) \int \cosh bx \cosh cx \, dx = \frac{1}{b^2 - c^2} (b \sinh bx \cosh cx - c \sinh cx \cosh bx) \quad (\text{para } b^2 \neq c^2)$$

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$$98) \int \cosh bx \sinh cx \, dx = \frac{1}{b^2 - c^2} (b \sinh bx \sinh cx - c \cosh bx \cosh cx) \quad (\text{para } b^2 \neq c^2)$$

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$$99) \int \sinh(ax + b) \sin(cx + d) \, dx = \frac{a}{a^2 + c^2} \cosh(ax + b) \sin(cx + d) - \frac{c}{a^2 + c^2} \sinh(ax + b) \cos(cx + d)$$

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$$100) \int \sinh(ax + b) \cos(cx + d) \, dx = \frac{a}{a^2 + c^2} \cosh(ax + b) \cos(cx + d) + \frac{c}{a^2 + c^2} \sinh(ax + b) \sin(cx + d)$$

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$$101) \int \cosh(ax + b) \sin(cx + d) \, dx = \frac{a}{a^2 + c^2} \sinh(ax + b) \sin(cx + d) - \frac{c}{a^2 + c^2} \cosh(ax + b) \cos(cx + d)$$

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$$102) \int \cosh(ax + b) \cos(cx + d) \, dx = \frac{a}{a^2 + c^2} \sinh(ax + b) \cos(cx + d) + \frac{c}{a^2 + c^2} \cosh(ax + b) \sin(cx + d)$$