

$$1. \int \sec^{-1} 3x^2 dx$$

$$\int \tan^{-1} \frac{1}{x^2} dx = \int \arctan \left(\frac{1}{x^2} \right) dx = \int \operatorname{arccot} (x^2) dx$$

$$= x \operatorname{arccot} (x^2) - \int \frac{2x^2}{x^4+1} dx$$

$$= \int \frac{2x^2}{x^4+1} dx = -2 \int \frac{x^2}{x^4+1} dx$$

$$= -2 \int \frac{x^2}{(x^2 \sqrt{2x+1})(x^4 \sqrt{2x+1})} dx$$

$$\frac{1}{2^{3/2}} \int \frac{x}{x^2 - \sqrt{2x+1}} dx - \frac{1}{2^{3/2}} \int \frac{x}{x^2 + \sqrt{2x+1}} dx$$

$$= \int \left(\frac{2x - \sqrt{2}}{2(x^2 - \sqrt{2x+1})} \right) + \frac{1}{\sqrt{2}} \frac{1}{(x^2 - \sqrt{2x+1})} dx$$

$$\frac{1}{2} \int \frac{2x + \sqrt{2}}{x^2 + \sqrt{2x+1}} + \frac{1}{\sqrt{2}} \int \frac{1}{x^2 - \sqrt{2x+1}}$$

$$= \ln \left(\frac{x^2 - \sqrt{2x+1}}{2} \right) + \operatorname{arctan} (\sqrt{2x+1})$$

$$= \ln x^2 + \sqrt{2x+1} + \operatorname{arctan} (\sqrt{2x+1})$$

$$= 2 \left(\frac{-\ln(x^2 + \sqrt{2x+1})}{2^{3/2}} \right) + \frac{(x^2 - \sqrt{2x+1})}{2^{3/2}}$$

$$= 2 \ln(x^2 + \sqrt{2x+1}) - 2 \ln(x^2 - \sqrt{2x+1}) - \operatorname{arctan} \left(\frac{2x + \sqrt{2}}{\sqrt{2}} \right)$$

$$- 4 \operatorname{arctan} \left(\frac{2x - \sqrt{2}}{\sqrt{2}} \right) - 2^{5/2} \operatorname{arctan} \left(\frac{1}{x^2} \right) x + c$$

$$2 \cot^{-1} \sqrt{2x} = \int \frac{2 \sqrt{2x}}{2x+1} dx$$

$$= \int \frac{\sqrt{2x}}{\sqrt{2x+1}} dx$$

$$= \frac{1}{2} \left[\sqrt{2x+1} + \frac{\sqrt{2x}}{\sqrt{2x+1}} \right] + C$$

3.

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \csc^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \sinh^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{\sqrt{a^2+x^2}} dx = \sinh^{-1} \frac{x}{a} + C$$

$$4. \int \cos^3 \frac{2x}{3} dx = \int \cos^2 \frac{2x}{3} \cdot \cos \frac{2x}{3} dx = \int (1 - \sin^2 \frac{2x}{3}) \cos \frac{2x}{3} dx$$

$$\int \cos \frac{2x}{3} dx - \int \sin^2 \frac{2x}{3} \cos \frac{2x}{3} dx$$

$$\sin \frac{2x}{3} - \int \sin^2 \frac{2x}{3} \cos \frac{2x}{3} dx$$

$$\sin \frac{2x}{3} - \left(\sin \frac{2x}{3} \right)^3 + C$$

$$\boxed{\sin \frac{2x}{3} - \frac{1}{3} \sin^3 \frac{2x}{3} + C}$$

$$5. \int \frac{\sec^4 2x dx}{\sec^2(2x) \sec(2x) dx}$$

$$\int (\tan^2(2x) + 1) \sec^2(2x) dx$$

$$\int v^2 \frac{dv}{2} + \int \tan^2 v dv = \frac{1}{2} \int v^2 dv + \int \sec^2 v dv$$

$$\frac{1}{2} \left(\frac{v^3}{3} \right) + \frac{1}{2} \tan v + C = \frac{1}{6} + \frac{1}{2} \tan v + C$$

$$\boxed{= \frac{1}{6} \tan^3(2x) + \frac{1}{2} (2x) + C}$$

$$6. \text{csc}^{-1} 2x^2 dx$$

$$2x^2 \cdot \text{csc}^{-1} \int x^{\frac{-1}{x\sqrt{x^2-1}}} dx du = \frac{1}{x\sqrt{x^2-1}} dx$$

$$2x^2 \cdot \text{csc}^{-1} \int \frac{-1}{\sqrt{x^2-1}} dx \quad \frac{\int du}{\sqrt{u^2-a^2}} = \ln |u + \sqrt{u^2-a^2}|$$

$$\boxed{2x^2 \cdot \text{csc}^{-1} \ln |x + \sqrt{x^2-1}| + c}$$

$$7. \text{cot}^{-1} \sqrt{2x} dx$$

$$\int \text{arc cot} \sqrt{2x} dx$$

$$v = \sqrt{2x}$$

$$v = (\sqrt{2x})^{1/2}$$

$$\frac{d}{dx} (\sqrt{2x})^{1/2} (2x)^{1/2} \frac{d}{dv} (2x)$$

$$(2x)^{1/2}$$

$$\frac{dv \sqrt{2}}{2} x^{-1/2} dx$$

$$x \text{arc cot} + c (\sqrt{2x}) + \sqrt{\frac{2}{2}} \cdot \sqrt{x} - \frac{1}{2} \arctan(\sqrt{2x})$$

$$8. - \frac{\text{Sen}^{-1} \sqrt{2x^2}}{\sqrt{2x^2}}$$

$$\frac{d}{dx} (2x^2)$$

$$I = (x) = x^n F'(x) = nx^{n-1}$$

$$\frac{1}{2} (2x^2)^{-1/2} \frac{d}{dx} (2x^2)$$

$$(2x^2)^{-1/2} \frac{d}{dx} (x^2)$$

$$2x^2 (2x^2)^{-1/2}$$

$$\boxed{x \arcsin(\sqrt{2x^2}) + \frac{\sqrt{2}}{2} \sqrt{1-2x^2} + C}$$

$$9. \text{Senh} \frac{1}{x^2} dx = \ln(ut \sqrt{cx^2+1})$$

$$\frac{S_1 \cdot h + S_2}{x^2} dx$$

$$\frac{am}{an} = amn$$

$$S_1 \cdot 175 x^{-2} dx$$

$$\boxed{1.1752 x^{-2} + C}$$