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INSTRUCCIONES: Resuelve de forma clara y correcta las siguientes ecuaciones.

(LOS NUMEROS -1 SON EXPONENTES; LOS NUMEROS DESPUES DE LAS FUNCIONES TRIGONOMETRICAS SON EXPOENTES Y LOS NUMEROS DESPUES DE LAS VRIABLES SON POTENCIAS)

1.- $\int \sin^{-1} 3x^2 dx$

The image shows a handwritten solution on grid paper for the integral $\int \sin^{-1} 3x^2 dx$. The student uses the substitution $v = \sin^{-1} 3x^2$, which leads to $dv = \frac{1}{\sqrt{1-9x^4}} \cdot 6x dx$. The student then rewrites the integral as $\int \sin^{-1} 3x^2 \cdot \frac{1}{6} \cdot 6x \cdot \frac{1}{\sqrt{1-9x^4}} dx$, which simplifies to $\frac{1}{6} \int \sin^{-1} 3x^2 \cdot \frac{6x}{\sqrt{1-9x^4}} dx$. The student then uses the identity $\sin^{-1} u = \frac{\pi}{2} - \cos^{-1} u$ to rewrite the integral as $\frac{1}{6} \int \left(\frac{\pi}{2} - \cos^{-1} 3x^2 \right) \cdot \frac{6x}{\sqrt{1-9x^4}} dx$. This is split into two integrals: $\frac{1}{6} \int \frac{\pi}{2} \cdot \frac{6x}{\sqrt{1-9x^4}} dx - \frac{1}{6} \int \cos^{-1} 3x^2 \cdot \frac{6x}{\sqrt{1-9x^4}} dx$. The first integral is $\frac{\pi}{2} \int \frac{x}{\sqrt{1-9x^4}} dx$. The student then uses the substitution $w = 1-9x^4$, $dw = -36x^3 dx$, which leads to $\frac{\pi}{2} \int \frac{x}{\sqrt{w}} \cdot \frac{dw}{-36x^3} = -\frac{\pi}{72} \int \frac{1}{x^2 \sqrt{w}} dw$. The student then uses the substitution $z = \sqrt{w}$, $dz = \frac{1}{2\sqrt{w}} dw$, which leads to $-\frac{\pi}{72} \int \frac{1}{x^2 \cdot 2z} \cdot 2z dz = -\frac{\pi}{72} \int \frac{1}{x^2} dz$. The student then uses the substitution $u = 1/x$, $du = -1/x^2 dx$, which leads to $-\frac{\pi}{72} \int du = -\frac{\pi}{72} u + C = -\frac{\pi}{72} \frac{1}{x} + C$. The second integral is $-\frac{1}{6} \int \cos^{-1} 3x^2 \cdot \frac{6x}{\sqrt{1-9x^4}} dx$. The student then uses the identity $\cos^{-1} u = \frac{\pi}{2} - \sin^{-1} u$ to rewrite the integral as $-\frac{1}{6} \int \left(\frac{\pi}{2} - \sin^{-1} 3x^2 \right) \cdot \frac{6x}{\sqrt{1-9x^4}} dx$. This is split into two integrals: $-\frac{1}{6} \int \frac{\pi}{2} \cdot \frac{6x}{\sqrt{1-9x^4}} dx + \frac{1}{6} \int \sin^{-1} 3x^2 \cdot \frac{6x}{\sqrt{1-9x^4}} dx$. The first integral is $-\frac{\pi}{2} \int \frac{x}{\sqrt{1-9x^4}} dx$. The student then uses the substitution $w = 1-9x^4$, $dw = -36x^3 dx$, which leads to $-\frac{\pi}{2} \int \frac{x}{\sqrt{w}} \cdot \frac{dw}{-36x^3} = \frac{\pi}{72} \int \frac{1}{x^2 \sqrt{w}} dw$. The student then uses the substitution $z = \sqrt{w}$, $dz = \frac{1}{2\sqrt{w}} dw$, which leads to $\frac{\pi}{72} \int \frac{1}{x^2 \cdot 2z} \cdot 2z dz = \frac{\pi}{72} \int \frac{1}{x^2} dz$. The student then uses the substitution $u = 1/x$, $du = -1/x^2 dx$, which leads to $\frac{\pi}{72} \int du = \frac{\pi}{72} u + C = \frac{\pi}{72} \frac{1}{x} + C$. The final answer is $-\frac{\pi}{72} \frac{1}{x} + C + \frac{\pi}{72} \frac{1}{x} + C = 2C$.

2.- $\int \cos^{-1} 5x dx$

②

$$\cos^{-1} 5x \quad dx \quad v = \frac{-1}{\sqrt{1-x^2}}$$

$$5x \cdot \cos^{-1} + \int \frac{1}{\sqrt{1-25x^2}} \cdot x \, dx$$

$$5x \cos^{-1} + \int (1-25x^2)^{-1/2} \cdot x \, dx$$

$$5x \cos^{-1} + \frac{1}{50} \int (1-25x^2)^{-1/2} \cdot 50x \, dx$$

$$5x \cos^{-1} + \frac{1}{50} \int \frac{(1-25x^2)}{-\frac{1}{2}+1} \cdot 50x \, dx$$

$$5x \cos^{-1} + \frac{1}{50} \int \frac{(1-25x^2)^{1/2}}{\frac{1}{2}} + C$$

$$5x \cos^{-1} + \frac{1}{50} \int (1-25x^2)^{1/2} + C$$

$$5x \cos^{-1} + \frac{2(+25x^2)^{1/2} + C}{50}$$

$$5x \cos^{-1} + \frac{(1+25x^2)^{1/2} + C}{25}$$

$$5x \cos^{-1} + \frac{(1+25x^2)^{1/2} + C}{25} //$$

3.- $\int \tan^{-1} 1/x^2 \, dx$

4.- $\int \cos^3 \frac{2x}{3} dx$

A)

$$\begin{aligned} \int \cos^3 \frac{2x}{3} dx &= \int \cos^2 \frac{2x}{3} \cdot \cos \frac{2x}{3} dx \\ &= \int (1 - \sin^2 \frac{2x}{3}) \cos \frac{2x}{3} dx \\ &= \int \cos \frac{2x}{3} dx - \int \sin^2 \frac{2x}{3} \cos \frac{2x}{3} dx \\ &= \sin \frac{2x}{3} - (\sin \frac{2x}{3})^3 + C \\ &\equiv \sin \frac{2x}{3} - \frac{1}{3} \sin^3 \frac{2x}{3} + C \end{aligned}$$

5.- $\int \sec^4 2x dx$

⑤

$$\begin{aligned} \int \sec^4 2x dx &= \sec^2(2x) \sec^2(2x) dx \\ &= \int (\tan^2(2x) + 1) \sec^2(2x) dx \\ &= \int \tan^2(2x) \sec^2(2x) dx + \int \sec^2(2x) dx \\ &= \int u^2 \frac{du}{2} + \int \sec^2 v \frac{dv}{2} = \frac{1}{2} \int u^2 du + \frac{1}{2} \int \sec^2 v dv \\ &= \frac{1}{2} \left(\frac{u^3}{3} \right) + \frac{1}{2} \tan v + C = \frac{1}{6} u^3 + \frac{1}{2} \tan v + C \\ &= \frac{1}{6} \tan^3(2x) + \frac{1}{2} \tan(2x) + C \end{aligned}$$

6.- $\int \csc^{-1} 2x^2 dx$

⑥

$$\int \csc^{-1} 2x^2 dx \quad v = \csc^{-1} u \quad \int u dv = uv - \int v du \quad dv = dx$$

$$= 2x^2 \cdot \csc^{-1} - \int x \frac{-1}{x\sqrt{x^2-1}} dx \cdot du = \frac{-1}{x\sqrt{x^2-1}} \quad v=x$$

$$= 2x^2 \cdot \csc^{-1} \int \frac{-1}{\sqrt{x^2-1}} dx \int \frac{du}{\sqrt{u^2-a^2}} = \ln|v+\sqrt{v^2-a^2}|$$

$$= \underline{2x^2 \cdot \csc^{-1} \ln|x+\sqrt{x^2-1}| + C}$$

7.- $\int \cot^{-1} \sqrt{2x} dx$

⑦

$$\int \cot^{-1} \sqrt{2x} dx$$

$$v = \sqrt{2x}$$

$$v = \sqrt{2x} \cdot 1/2$$

$$\frac{d}{dx} (\sqrt{2x}) = 1/2 (2x)^{-1/2} \frac{d}{dv} (2x)$$

$$(2x)^{-1/2}$$

$$dv \sqrt{\frac{2}{2}} x^{-1/2} dx$$

$$x \operatorname{arccot} c \sqrt{2x} + \frac{\sqrt{2}}{2} \sqrt{x} + 1/2 \operatorname{arctan} \frac{(\sqrt{2x})}{c}$$

8.- $\int \sin^{-1} \sqrt{2x^2} dx$

⑧

$$\sin^{-1} \sqrt{2x^2} \quad v = \sqrt{2x^2}$$

$$\frac{d}{dx} (\sqrt{2x^2})$$

$$F(x) = x^n \quad f'(x) = n x^{n-1}$$

$$\frac{1}{2} (2x^2)^{-1/2} \frac{d}{dx} (2x^2)$$

$$(2x^2)^{-1/2} \frac{d}{dx} (x^2)$$

$$2x (2x^2)^{-1/2} \frac{d}{dx} \sqrt{2} dx$$

$$\underline{x \arcsin(\sqrt{2x^2}) + \sqrt{\frac{3}{2}} \sqrt{1-2x^2} + C}$$

9.- $\int \sinh \frac{1}{x^2} dx$

⑨

$$\sinh \frac{1}{x^2} dx = \ln(u + \sqrt{cx^2 + 1})$$

$$\int \frac{1 \cdot 1752}{x^2} dx \quad \frac{am}{an} = a \cdot m \cdot n$$

$$\int 1752 x^{-2} dx \quad m =$$

$$\boxed{11 \cdot 1752 x^{-2} + C}$$

